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RESEARCH ARTICLE

Omicron Waves in Few European Countries till June 2022 Modeled by a New Version of a Tracking Approach with Successive and Superimposed Waves

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Abstract: Our objective is a mathematical modeling in retrospective of the COVID-19 Omicron waves in the UK, France and Germany till mid June 2022. The aim is two-fold: ensure a good reproduction of the data with consistent parameters, also by comparing the results to the ones from an earlier study for the USA, and check the usefulness of a new, improved version of a recently published model. The main novelty of the approach used is the dynamical tracking of successive generations of infected people instead of treating the evolution of few large compartments within which the total population is partitioned. Because of the stronger transmission of Omicron, its waves start to dominate the Pandemic, and then the new model can be easily employed. The formalism is improved by employing better conditions for continuity when interconnecting solutions of differential equations and a superposition of waves related to independent pathogens. The daily observed new infection cases are described over a large time scale in a reasonable way after normalization, with deviations and differences due to country-specific factors. The time-position of the first calculated Pandemic peaks indicates a transition from the third to the fourth generation of infected people. The derived infection and recovery rates are consistent with those deduced for the USA. A correlation exists between initialization of relaxing restrictions and begin of a new wave or simply a jump up of the data locally. However, very often it happens nearby that a new independent wave emerges related to a different variant of the pathogen. Another important result is that describing in a reasonable way Epidemics by using consecutive waves and superposition of waves caused by different pathogen variants opens the possibility to investigate the COVID-19 Pandemic in its full time range, since early 2020 to present. In the future, we intend to work on that problem to obtain additional useful information on that particular Pandemic in some country (or region) and to develop further the model (and software) toward readiness to meet next possible challenges when they come.

Introduction

By about mid of 2022, the importance of the COVID-19 Pandemic's issue was not needing any special argumentation¹. Since the first signals for the seriousness of that Pandemic worldwide in the beginning of 2020², much has been learned about the SARS-CoV-2 virus itself, though this process is not finished because new virus variants/mutations appear with time, with different properties, including much increased contagiousness sometimes. This is the case with the Omicron variant identified in late November 2021³. Later, other sub-variants of it were found responsible for provoking new (or secondary) "waves" spreading out even faster and whose peaks lie at varying time intervals further away from the primary peak, but sometimes also very close^{4,5}. As matter of fact the first Omicron variant which became of importance (labeled shortly BA.1 in the literature) and the sub-variant labeled BA.2 which became dominant since February 2022 worldwide, were detected nearly simultaneously in South Africa and Botswana (joined also by BA.3). The present work is concerned with some mathematical aspects of addressing and modeling the Omicron-stage of the Pandemic⁶ in retrospective. However, one should never forget that dealing with the development of Epidemics is a multidisciplinary subject. Epidemiologists, specialists in infections, virologists, medics with different specializations, pharmaceuticals, immunologists, molecular biologists etc. naturally participate by addressing their relevant specialized issues. Thus, the modeling remains just a more or less reliable quest for mathematical apparatus able for short and/or long term forecasting or a posteriori description. In this way, it is possible to address more efficiently the issue of the health, social and economic price that societies have to pay to stop/control the Pandemic. Very recently, during the finalization of the present work in May 2023, the WHO declared⁷ that the COVID-19 pandemic "is now an established and ongoing health issue which no longer constitutes a public health emergency of international concern."

A bit more than an year earlier, in the beginning of March 2022, the Omicron wave has reached in many countries a maximum and decreasing trends were observed. Therefore the question of lifting different restrictions then raised again. At that time, there were objective and real preconditions which have been met and which have made reasonable such a program of "return to the normal." However, this general trend cannot be a basis for automatism in dealing with the problem, within about the same nature and tempo of lifting

restrictions everywhere. On the contrary, this should be done with taking into account the specific features of the situation at every place (country or even region, may be). Later, new developments occurred, to some extent related to the initialized processes of lifting restriction, but mainly to the spread of new sub-variants of Omicron^{4,5} described as being even more contagious than the original one. The present work, on the basis of improvements of the tracking model published recently in Ref.⁸ aims to reproduce in retrospective the data on the new daily infection cases in three European countries: the UK, Germany and France till June 2022. The improvements compared to Ref.⁸, consist in more continuous interconnection between consecutive waves and superposition of the waves related to pathogens with different properties. The paper is built as follows. After the introduction, the method used for the calculations is presented including the newly incorporated and improved features. Then, the results for the three countries are presented, and later discussed and compared, also to similar data for the USA. Finally, the conclusions of the present study are given.

The improved method employed for the calculations

A possibility to describe an Epidemics over the full time range of its duration is very suitable, indeed. However, this may be difficult for cases where the Epidemics becomes endemic, its pathogens undergo evolution or immunity (acquired by recovering from illness or given by vaccination) has a limited duration. To solve such problems, if possible, a class of involved Epidemics models, based so far mainly on Ref.⁹, were developed and applied, including also many further additional features incorporated and attempts to take into account more complex effects that originally done in.⁹ Because their basic idea is to partition the population into compartments (groups), these models are commonly denoted as "compartmental" ones. Following the logic of the infection propagation, transition rates describe the probability for individuals to move from one compartment to another. Thus, within the simplest version of the models considered, namely the so-called SIR model, there exist three compartments consisting of the people susceptible to infection (S), the infected people (I) and the number of the recovered ones (R). The number of people in each of them is a function of the time t . Other versions upgrading the SIR approach contain more compartments including people with other, different status in the propagation of the infection and the corresponding transition rates. Thereby, in

some cases an immunity with limited duration is also considered. This seems to be the case of the Covid-19 Pandemic, at least for a part of the population.

Recently, we published ⁸ a new and simple method for describing Epidemics including consecutive waves. This method uses also a kind of compartmentalization, where one of the two main structures is the group of infected people decomposed in turn into the different generations of infected individuals appearing naturally in the chain of the “human to human to human ...” transmission. These generations are tracked as well as the whole process of the infection propagation. This feature represents the most important novelty of the model. Such tracking is not possible within the SIR-like models. The other main structure are the generations of recovered people who leave the corresponding generation of infected individuals after some interval of time needed for recovery (see below). In the model, linear differential equations are solved ⁸ while e.g. the SIR-like models require the solving of non-linear differential equations. The processes of infection spread and recovery are governed by two transition rates, λ_R (rate of recovery) and λ_C (rate of infection or spreading rate). The rates are assumed to be constant in time when a single Epidemics wave is described. Here, one comes to one of the limitations of the model, namely that the infection spread is supposed to occur, obeying statistical laws, within large enough population which provides an inexhaustible source of people susceptible to infection. Simultaneously, the time for recovery is treated as a random variable deciding the “fate” of each infected single individual. Similar to most of the models describing Epidemics, the distribution of the recovery times represents an exponential distribution with mean recovery rate $\lambda_R = 1/\tau_R$ (with τ_R being the mean time needed for recovery). Concerning the rate λ_C of the infection spreading, it is also considered to remain constant for a given Epidemics wave. However, it may be very largely influenced by measures as confinement, reduction of social contacts, reconfinement, lifting restrictions, appearance of new virus variants etc. and then another consecutive wave has to be included in the considerations as done e.g. in Ref. ⁸. Studies of these effects are presented in Refs. ¹⁰⁻¹² and references therein.

As already mentioned in the Introduction, the Omicron variant of SARS-CoV-2 is characterized by higher contagiousness than the other dominant previous variants (in different time periods), the δ -one “possessing” the previous record. Although in

some countries mixing of the δ and Omicron waves was observed by mid January 2022, in others a very strong effect of increase of the daily new cases indicated the forthcoming dominance of Omicron. The very simple mathematics of Ref. ⁸ opens the possibility to forecast the development of Epidemics in time for different infection and recovery rates, in particular in isolated systems. This is quite relevant for getting a fast idea on cases related to the appearance of strongly contagious infection pathogens as Omicron and the considerations in that model used in the present work are not limited to the time of the Epidemics outbreak. The main point is, as it is in many scientific disciplines, that when one factor dominates all other factors which influence somehow a phenomenon, scientists consider such a case as a testing ground for checking a particular model and simultaneously learn more about the overwhelmingly dominant factor. Simply the picture at an early stage is quite pure and not obscured by later developments which are likely to occur because of some desynchronizations in the otherwise globalized world. However, the development of the Pandemic after March 2022 indicated the appearance of a new increase of the new infection cases per day which was associated with the impact of new sub-variants of Omicron. While a simple lifting of some restrictions would lead to a new consecutive wave with somewhat larger infection rate λ_C , the effect of a pathogen with different properties should be rather considered as a new, independent wave which is superimposed on the first Omicron one. Therefore, within the present work the model ⁸ was further developed as discussed below.

Let us remind the basic features of the new tracking method presented originally in Ref. ⁸. The first step is the determination of the functions $i_n(t)$ which represent the number of infected individuals belonging to the different generations enumerated by n . The logic is borrowed from the description of the time evolution of the population of excited states in nuclear, atomic and molecular physics which are interconnected by transitions (mainly electromagnetic, photons and γ -quanta). The main difference with the case discussed in the present work (and Ref. ⁸) is that while in the analog situations many levels with very different deexciting transition probabilities each are under consideration, here one has successive generations of infected individuals each one populated by the previous generation with probability per unit time (rate) λ_C and undergoing recovery with the rate λ_R . Hence a modification of the formalism is needed (see).⁸ Namely, the first generation (the

so-called zero patients) $i_1(t)$ simply obeys a differential equation similar to the radioactivity decay law of Rutherford-Soddy:

$$\frac{di_1(t)}{dt} = -\lambda_R i_1(t) \quad (1)$$

with the solution $i_1(t) = \lambda_R i_1(0) e^{-\lambda_R t}$. For the next generations, the linear differential equation for the derivative of $i_n(t)$ contains on the r.h.s. the decrease of $i_n(t)$ and the increase due to infections from the previous generation i_{n-1} , always with the same rates i.e:

$$\frac{di_n(t)}{dt} = -\lambda_R i_n(t) + \lambda_C i_{n-1}(t) \quad (2)$$

The general solution (result) for the n^{th} generation reads⁸ :

$$i_n(t) = \frac{\lambda_C^{n-1} i_1(0) t^{n-1} e^{-\lambda_R t}}{(n-1)!} \quad (3)$$

The total number of infected people as function of time is given by the sum over the different generations:

$$i(t) = \sum_{k=1,2,\dots,N_{max}} \frac{\left(\frac{\lambda_C}{\lambda_R}\right)^{k-1} n_1(0) x^{k-1} e^{-x}}{(k-1)!} \quad (4)$$

where N_{max} is the last generation appreciably populated at time t and $x = \lambda_R t$.

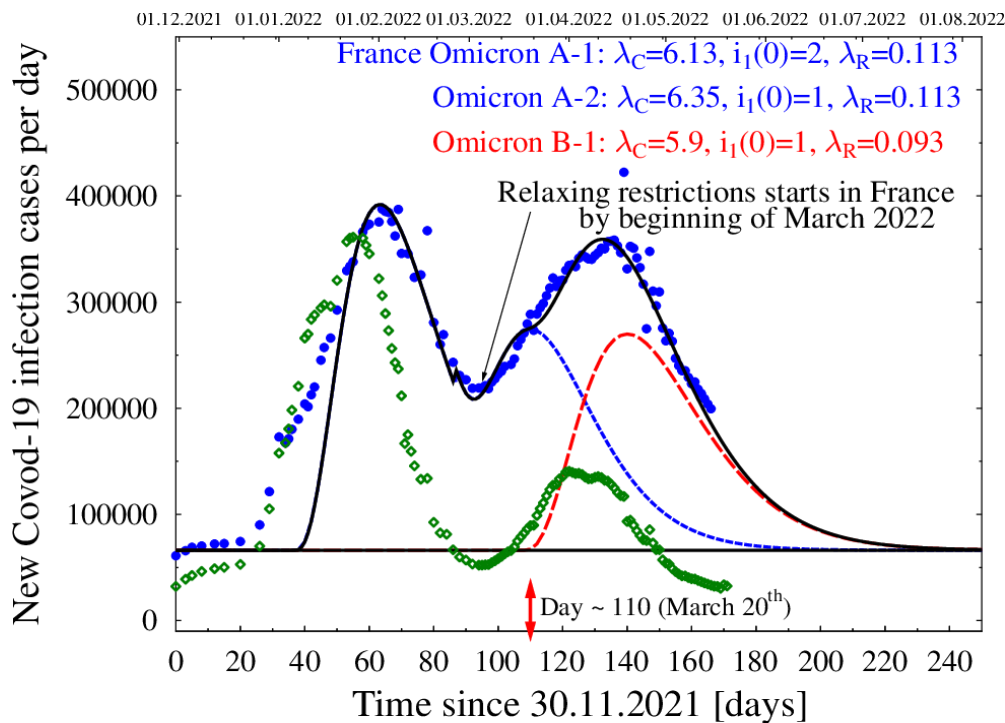


Figure 1. The Omicron waves of the COVID-19 Pandemic in France. All data were taken from the site.¹³ The original raw data on the newly registered cases per day and the corresponding normalized data points are represented by open diamonds and filled circles, respectively. The plateau-like part of the data before the effect of the first sharp increase is considered as a constant (averaged) background and reflects the previous stage of the Pandemic. The data are interpreted as a superposition of two independent waves A and B generated by the original Omicron variant and the late sub-variants of it, respectively (cf. Eq.12 and the continuous curve in the figure). The wave A as matter of fact consists of two consecutive waves, the second one (labeled as A-2) obviously starting to develop after the initialization of lifting restrictions. The reproduction of A-1 and A-2 is based on calculations using Eq.11 and Eq.12 with the results illustrated by a short-dashed line. For B-1, the expression for $i_n(t)$ given by Eq.3 and the expression in Eq.7 for $C(t)$ were used and the best result is represented by a long-dashed line. The superposition of the waves A-1,A-2 on the one hand and B-1 on the other hand is characterized by $a=0.34$ and $b=0.66$. The parameters derived are also indicated. The infection and recovery rates are in days⁻¹. See also text.

The expression resembles a sum of Poisson distributions weighted by the factors $\left(\frac{\lambda_C}{\lambda_R}\right)^{k-1}$ and

overall scaled by the factor $i_1(0)$. This formulation may be not fully correct from a mathematical point of view because of the discrete character of the

Poisson distribution and therefore the resemblance is rather formal. The quantity defined as $R_0 = \lambda_C/\lambda_R$ in the framework of the present model⁸ is the analog of the so-called basic reproductive number. This reproductive number, though widely used, is somewhat model-dependent as discussed in detail in Refs.¹⁴⁻¹⁷. Roughly, this is the number of people who will be infected by one contagious individual up to the moment when this individual completely recovers from the infection. It is easy to see the complexity of R_0 by considering the fact that the infection spread depends not only on the transmission probability but also on the frequency of contacts e.t.c. The quantity R_0 is thoroughly discussed in Ref.⁸ (starting around Eq.15 in Sec.2 of that work and later, when the results in Table 1 there are discussed). Whatever it is, Eq.3 can be rewritten as

$$i_n(t) = (R_0^{PW})^{n-1} \frac{i_1(0)(\lambda_R t)^{n-1} e^{-\lambda_R t}}{(n-1)!}. \quad (5)$$

where the superscript "PW" indicates that the numerical value of that quantity is relevant exclusively to the present work (and model). It is to be noted that in the formalism presented the dependence on R_0^{PW} is very strong, changes with the generation number n and is different from that in the other models designed so far to describe Epidemics in time.

A more careful examination allows to conclude that the peak (maximal) value of $i_{N_{max}}(t)$ dominates the sum of all generations of infected people $i_n(t)$ up to N_{max} . The only maximum of $i_n(t)$ occurs at

$$t_{max} = \frac{(n-1)}{\lambda_R} = (n-1)\tau_R, \quad (6)$$

a relation which is very useful to estimate n provided that τ_R is known. If one considers $i_n(t)$ as a distribution over time, its expectation (mean) value $M1 = n\tau_R$ lies on the time axis later (by τ_R) than the maximum of $i_n(t)$. Thus, the expected time range of the epidemics depends on τ_R and n , and increases with both of them, of course. The most often used data for monitoring Epidemics consists of the sequence in time of newly infected

$$i'_n(y) = D_n i_1(0) \frac{\left(\frac{\lambda_{CII}}{\lambda_R}\right)^{n-1} (\lambda_R y)^{n-1} e^{-\lambda_R y}}{(n-1)!} + \sum_{l=0}^n i_1(0)(\lambda_R)^l (\lambda_{CII}(1 - e^{-\alpha y}) + \lambda_C e^{-\alpha y}) e^{-\lambda_R(y+T)} C_{nl}(T+y)^l \quad (9)$$

Here, the coefficients D_n and C_{nl} have to be determined by solving the system of differential equations in time region II with the condition for

people per day, normally established by tests. It can be calculated as

$$C(t) = \lambda_C \sum_{n=2}^{K_{max}} i_{n-1}(t). \quad (7)$$

The upper limit K_{max} within the generation numbers depends on the Epidemics status at time t . When a particular data set is considered K_{max} is treated as a discrete fitting parameter ensuring the best description as explained in Ref.⁸.

The data on the new infection cases per day after end of March 2022 in the countries considered display new peaks ("waves") lying close in time with respect to the first Omicron peak. Their interpretation seems to be very complicated. It can be made more easier if a formalism interconnecting the consecutive waves is used by analogy to what was proposed in Ref.⁸. In the present work, an attempt to improve the interconnection is made for two consecutive waves. The main improvement is that when solving the underlying linear differential equations in the two time regions (I and II, those of the first and second wave, respectively) a more smooth transition between the solutions is accomplished. Thereby, not only equal functional values at the time of initiation of the second wave are ensured, but also equal values of their first time derivatives. For this purpose, the following considerations are made.

Let us denote by y the time difference between the global time t and the moment of the initiation of the second wave T , i.e. $y=t-T$. The differential equation, analog to Eq.2, for the number of the new daily infections i'_n in region II reads

$$\frac{di'_n(y)}{dy} = -\lambda_R i'_n(y) + \lambda_{CII} i'_{n-1}(y) + \lambda_C i_{n-1}(T+y) \quad (8)$$

The last term in the equation describes the infections from the $n-1$ generation from time region I whose existence continues in region II, indeed. By analogy with Eqs.3,4, one can assume that the general solution of Eq.8 is given by:

continuous interconnection of the first time derivatives with those in region I, too. The time-

dependent term $(\lambda_{C_{II}}(1 - e^{-\alpha y}) + \lambda_C e^{-\alpha y})$ ensures the smooth interconnection and simulates to some extent the effect that the second wave does not abruptly occurs but there is some transitional period determined by the time constant α (the larger α , the shorter the transition period). By replacing the assumed general solution given by Eq.9 in Eq.8 one obtains a moderately

complicated expression which can be compared to the result of direct differentiation of $i'_n(y)$ as expressed by Eq.9. Then, applying the strategy to compare terms with equal powers of the variable y on both sides of the identity equation, one can determine D_n and C_{nl} via recursive formulae. Thus, one obtains that $D_n = D_{n-1}$, $C_{nn} = 0$, $C_{n-1l} = 0$ and the general formula for C_{nl} which reads

$$C_{nl} = \frac{(\lambda_C)^{n-1} T^{n-l-1}}{(\lambda_R)^l (n-2)! (\alpha(\lambda_{C_{II}} - \lambda_C)T - \lambda_{C_{II}}\lambda_C T + \lambda_C l)}. \quad (10)$$

Replacing the result for C_{nl} in Eq.9 ends up with the final result

$$i'_n(y) = i_1(0) \frac{\left(\frac{\lambda_{C_{II}}}{\lambda_R}\right)^{n-1} (\lambda_R y)^{n-1} e^{-\lambda_R y}}{(n-1)!} + \frac{i_1(0)(\lambda_{C_{II}}(1 - e^{-\alpha y}) + \lambda_C e^{-\alpha y})e^{-\lambda_R(y+T)}}{(n-2)!} \times \sum_{l=0}^{n-2} \frac{(\lambda_C)^{n-1} T^{n-l-1} (T+y)^l}{(\alpha(\lambda_{C_{II}} - \lambda_C)T - \lambda_{C_{II}}\lambda_C T + \lambda_C l)}. \quad (11)$$

This result improves the findings of Ref.⁸ with respect to interconnecting consecutive waves (Eq.33 in that work) by requiring continuity of the functions $i_n(t)$. It has to be noted the function $C(t)$ describing the full number of new infection cases per day may display some discontinuity features at the interconnection because of the different infection rates λ_C and $\lambda_{C_{II}}$ multiplying the $i_n(t)$ functions in the time regions I and II (i.e. on both sides of T).

Another improvement (or novelty) in the present work is the use of superposition of waves associated with different virus variants or sub-variants. Since the pathogens differ, these waves caused by the latter can be considered as independent and adding up to yield the total number of new infection cases. At this stage, the weight factors of the independent waves are treated as adjustable parameters, constant in time. In practice, however, the weighting factors undergo an evolution in time ending up with one dominant wave. This effect remains to be implemented in future versions of the approach, also when realistic data on this issue will be more readily available. Formally, the superposition is expressed as

$$C_{total}(t) = aC_A(t) + bC_B(t) \quad (12)$$

where $a+b=1$. In the context of the COVID-19 Pandemic till about June 2022, the wave "A" may be associated with the original Omicron variant of the SARS-CoV-2 virus (labeled shortly as BA.1)

while the wave "B" is related to sub-variants dominated by the BA.2 one.

The relevant data sets of points for the UK, France and Germany were taken from the site¹³ and are displayed in Figs. 1,2,3 with open diamonds. They cover the time range from 30.11.2021 to about 20.05.2022. As discussed in Ref.⁸, the total population is, of course, not tested every new day. Also, the number of people tested fluctuates each day, sometimes drastically (e.g. on weekends or great Holidays). A more realistic picture may be obtained using data rolled (smoothed) on the basis of 7 or 14 days data. It is much better, however, to perform a normalization of the above raw data for following more precisely in time and compare the absolute number of new daily cases of infections for the same constant fraction of the total population. There are factors, however, which may bias the normalization as e.g. possible concentration of tests in regions of the country with higher number of new cases or within clusters of enhanced transmission, several tests for one individual in the period considered, the very nature and sensitivity of the tests as diagnostic tools e.t.c., and these factors may vary with time. The explicit use of the normalization discussed above is another novelty in the approach of Ref.⁸ to describe/reproduce the new infection cases per day.

Results on the daily new cases of infections

The data points for France, the UK and Germany presented in Figs.1,2 and 3, respectively, display a quite complicated behavior with time, both for

raw data and normalized points. Since the present study is dedicated to the Omicron waves, no attempt was made to describe the data before the appearance of a clear signal for very sharp increase of the new daily infection cases.

Therefore, we decided to take into account the contribution of the Pandemic before Omicron by subtracting a constant background determined by averaging over a subset of points with similar time-behavior starting from December 2021

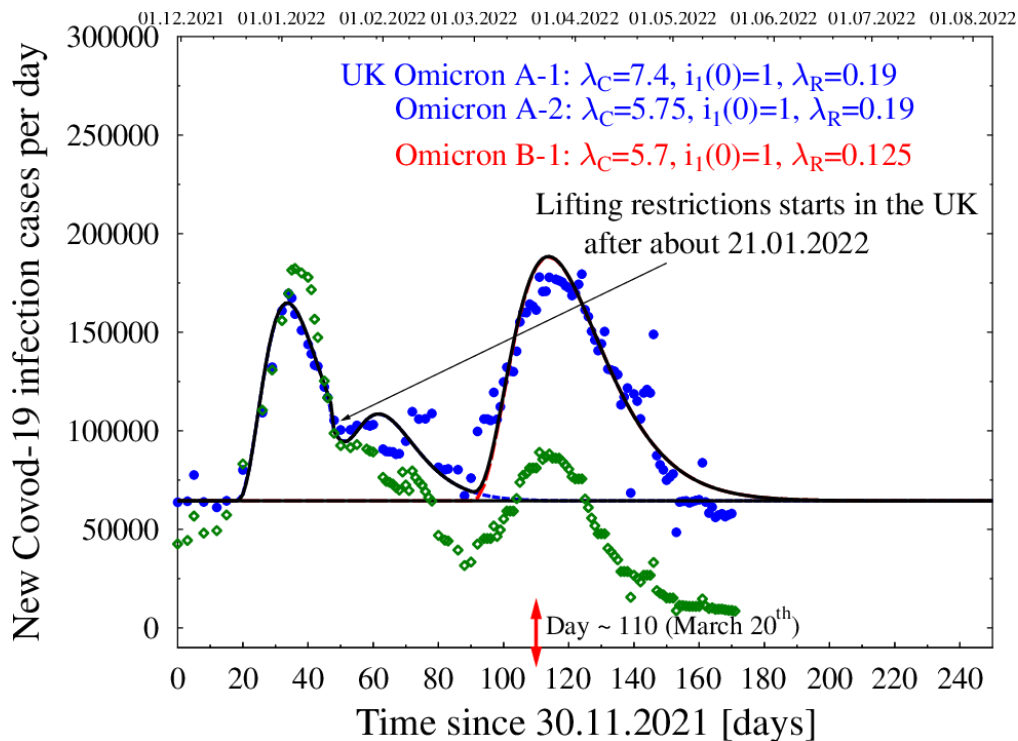


Figure 2. The same as in Fig.1 but for the UK. The superposition of the waves A-1,A-2 on the one hand and B-1 on the other hand is characterized by $a=0.05$ and $b=0.95$. See also text.

In this way, the beginning of January 2022 is characterized by an effect of increase which is definitely observed. In France, however, a more pure Omicron effect is observed later, preceded by a period where there is some admixed role of earlier variants (the δ one on top, of course) and the incoming Omicron wave. Some delay is observed also in Germany. Thus, the time position of the start of the Omicron wave in each country was treated as an adjustable parameter allowing a reproduction of the fast increase accompanied by a change of the slope of splines passing through a set of successive data points toward higher value. After that, calculations according to the approach presented in the previous section were performed in order to reproduce the data. For this purpose, the function $C_{total}(t)$ was used to fit the normalized data in Figs.1,2 and 3 for the specified cases. Thereby, the parameters λ_C , λ_R and $i_1(0)$ were the main ones to be varied for each of the three waves (the last of which being related to a new pathogen, another sub-variant of the Omicron virus). The fitting problem is simplified to some extent because the first time region I can be considered nearly independently

of what happens next. However, when the second wave A-2 starts at time T , some adjustments had normally to be done for better description of the data points around T . Other adjustable parameters are the starting days of the waves whose position is approximately clear but a fine tuning is necessary to find the start-times ensuring the best results as well as the weighting factors a and b . In the next sections, we consider specifically the data and quality of their reproduction for the three countries considered.

France

All data are normalized to the point at 03.01.2022 (both earlier or later registered) and are displayed in Fig.1 with filled circles. At this date, 1147160 tests were performed in France¹³. The peak position of the fitted curve for the first Omicron wave (A-1) is about February 4th (day 66 since the global start of that wave). This corresponds roughly to 29 days after the “start” of the Omicron wave in France at day 37 and according to the relation $\Delta t_{max} = (n - 1)\tau_R$ (cf. Eq.6), with $\tau_R = 1/\lambda_R \approx 9$ days, this indicates that $n=4$ i.e. this is the moment when the fourth

generation of infected people is dominant (one should not forget that this generation provokes infections in the 5th generation too). The second wave A-2 starts about the moment when the relaxing of the restrictions starts and at first glance this may be the reason for its initialization, with slightly higher λ_{CII} as indicated in Fig.1. The admixing wave B-1 is calculated to start about day 108 i.e. March 18th. However, the BA.2 sub-variant of the Omicron virus is supposed to be dominant in Europe by February 2022 so the fitting procedure may not very precisely disentangle the positioning in time of B-1 and A-2. Yet, it is highly probable that both factors are responsible for the observed behavior of the new daily infection case since beginning of March and later.

The derived parameters, with some of them indicated in the figure, will be discussed together with the results for the UK and Germany at the end of the present main section.

The UK

The analysis presented in Fig.2 is similar to the analysis for France in Fig. 1, but the data look

quite different. The normalization is made with respect to the data point at 03.01.2022 when 1515530 tests were performed. An important specific feature is a very fast increase of the first Omicron wave with the largest λ_C among all cases considered in the present work. Other fitted parameters are also indicated in Fig.2. The importance of the effect of lifting restrictions, which was initiated about 21.01.2022, is very clearly seen in the UK. Large fluctuations of the new infection cases per day are observed both in raw and normalized data although in average the decreasing trend is conserved and the lifting of restrictions finally seems to be made not too early. The admixing wave B-1 start sufficiently late to form a well distinguished and separated peak. It has also to be mentioned that the Omicron wave has started a bit earlier in the UK compared to Germany and France.

The derived parameters, with some of them indicated in the figure, will be discussed together with the results for the Germany and France at the end of the present main section.

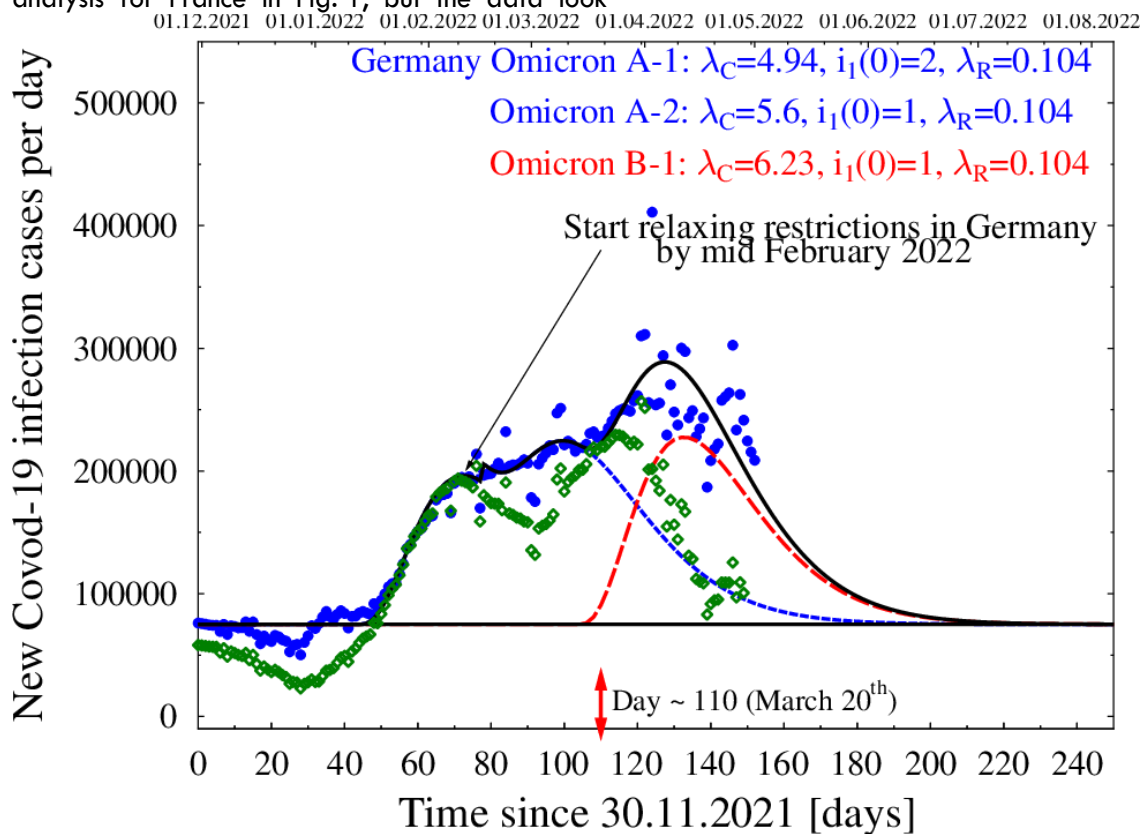


Figure 3. The same as in Fig. 1 but for Germany. Here, the maximum of the first peak is predicted about February 12th. A vertical line indicates the date 14.02.2022 when a three-step relaxing restriction did start and few days later the expected decreasing trend was suddenly transformed in an opposite, increasing one mixed with the new wave B-1. The superposition of the waves A-1,A-2 on the one hand and B-1 on the other hand is characterized by $a=0.5$ and $b=0.5$. See also text.

Germany

The analysis presented in Fig.3 in very general terms is similar to what has been shown in Figs.1,2 for France and the UK, respectively. The similarity is mainly due to the decomposition into the same number of Pandemic waves. However, there are features and differences in the derived parameters which need some special comments. First, the peak structures in the case of Germany are strongly overlapping. This makes the analysis more difficult and there is a larger uncertainty in the positioning of the waves A-2 and B-1 within the fitting procedure. The normalization in Fig.3 for Germany was made with respect to the data point at 30.01.2022 when 364841 tests were performed. Concerning normalization, it is worth to mention that the drop of the new daily infection cases around Christmas 2021 cannot be completely corrected even on a day per day normalization basis. Also, large fluctuations in the behavior of the normalized points are observed in the vicinity of the maximal (peak) values. This may be an indication that for the specific case of Germany the proposed way of normalization within the model is biased by some factors which remain to be investigated. These effects seem to be related to the fact that the number of daily tests is not much larger than the confirmed cases by them, each day. A three-step relaxing restrictions process did start about mid of February, but few days later a new increasing trend to even higher values occurred. This may suggest that the process of lifting restrictions began a bit too early. However, about February 12th information appeared that a new sub-variant of Omicron (the BA.2 one) spreads already with very high rates. This coincidence indicates how difficult is to disentangle the waves of different character within the complete data set on the new daily cases of infection.

Discussion

The parameters of the best fitting curves in Figs. 1,2 and 3 are summarized in Table 1. First of all, the infection rates $\lambda_C = 4.94 \div 7.4 \text{ days}^{-1}$ are very large. They can be compared to the value of 0.71 days^{-1} derived in Ref.⁸ for the first wave in the USA and Europe in spring of 2020 caused by the original Wuhan virus. In Table 3 of the latter

work, the rates of transmission are compared in relative units with respect to contagiousness (i.e. for the Wuhan virus one has 1.0, for $\alpha : 1.4 \div 1.9$, for $\delta : \geq 3$). In addition, in calculations according to the model of Ref.⁸ the ratio λ_C/λ_R participates with ever increasing effect, at the power of $(n-1)$ for every next generation number n . Therefore there is no wonder that the Omicron wave became so fast the dominant one in the time-period considered. For larger τ_R (smaller λ_R) the effect is complementary enhanced.

In addition, there are indications that the decreasing part of the data after the peak maxima can be also well described after normalization by the calculations. The need of normalization is related to the gradual decrease of the daily tests in most of the countries once the peak value has been reached. This is done with the expectation that in general, after the maximum only a decrease may be expected. However, a realistic estimate of its actual speed of spread and status (i.e. when a sufficiently low level will be reached) is of importance for the control of the Pandemic and eventually lifting restrictions, for example.

On the other hand, effects as massive social events, relaxation of restrictions or involvement on new, more contagious various variants modify the decreasing trend, sometimes initiating new waves. The use of normalization provides a stronger signal for such effects when the data are displayed as function of time compared to the raw data. At an earlier stage of the first Omicron wave, the evolution downwards after the peak has been related in the literature to the existence of a very large fraction of the population possessing protection against Covid-19 due to vaccination as well as to natural immunity acquired after recovery from the illness, both being of somewhat temporary character and with a duration that may be different for different individuals. The UK was the first country where such effects have been discussed and intentions of lifting gradually all restriction measures were declared and taken, followed soon by others. In this context, the example and the experience of the development in the UK may be very useful (cf. Fig.2)

Table 1. Summary of parameters derived by fitting the data on the new daily infection cases for the countries considered in the present work. The initial date of the first Omicron wave (A-1), which is treated as an adjustable parameter, is displayed in the second column. On top is the number of days elapsed since December 1th 2021 while below it the calendar day (in format DD.MM) in 2022 is shown (for the UK, the calendar date is in 2021). The next two columns present the parameters used to calculate the different generations of infected people $i_n(t)$ (λ_C and λ_R) which fit best the data. All rates are in days⁻¹. The mean recovery time τ_R is shown in column 5 (in days). The next column 6 presents the starting date of the second wave A-2 related most probably to relaxation of Pandemic restrictions (in days) followed by its new infection rate ($\lambda_{CII} \neq \lambda_C$). In column 8, the initial date of the independent admixing wave B-1 is shown followed in the next two columns by its parameters λ_C^B and λ_R^B , respectively. See also text.

Country	A-1	λ_C^A	λ_R^A	τ_R^A	A-2	λ_{CII}^A	B-1	λ_C^B	λ_R^B	τ_R^B
	day				day		day			
France	37	6.13	0.1132	8.8	50	6.35	108	5.9	0.093	10.8
	06.01				29.01		18.03			
UK	18	7.4	0.19	5.3	30	5.75	90	5.7	0.125	8
	18.12				30.12		28.02			
Germany	44	4.94	0.104	9.6	78	5.6	104	6.23	0.104	9.6
	13.01				16.02		14.03			
USA	32	7.3	0.165	6.6	90	6.05	144	6.7	0.135	7.4
	01.01				28.02		12.04			

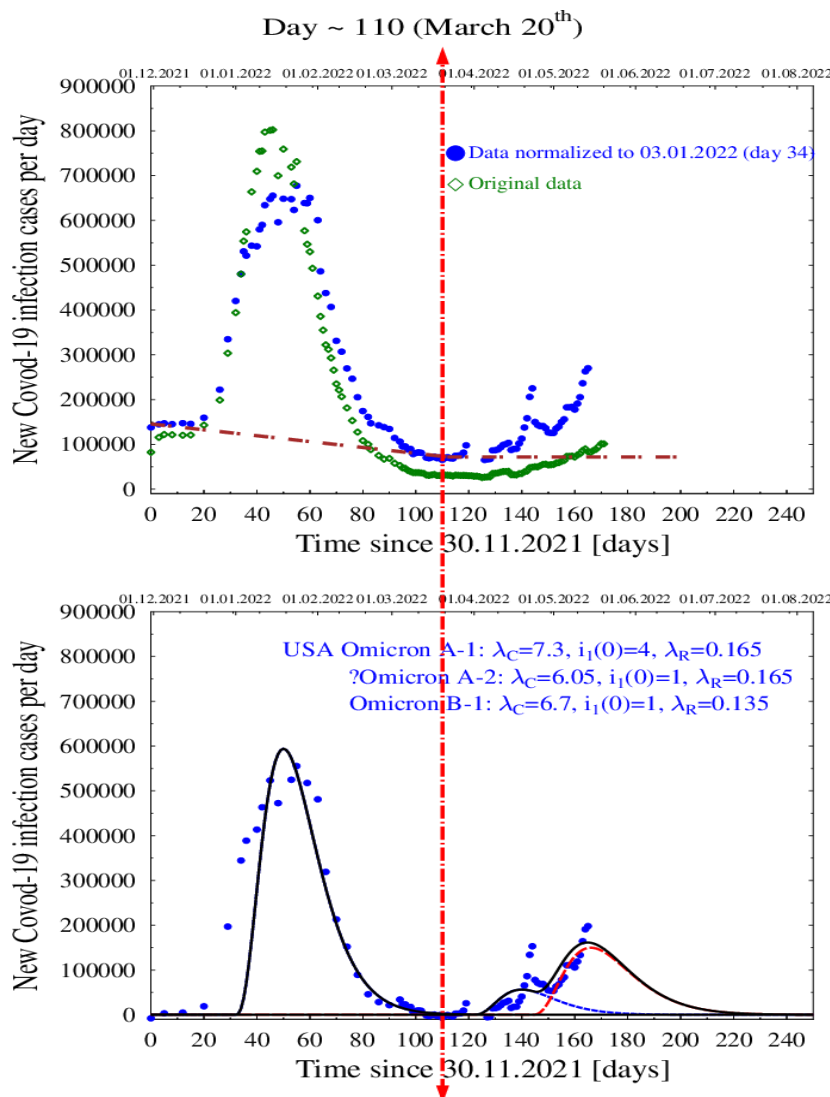


Figure 4. Top panel: The Omicron waves of the COVID-19 Pandemic in the USA. The data were taken from the site.¹³ The original raw data on the newly registered cases per day and the corresponding normalized data points are represented by open diamonds and filled circles, respectively. Obviously, the assumption for a constant ‘background’ is not fulfilled out of the peak zones. Therefore, a dedicated background subtraction was performed as explained in the text. The result of that subtraction from the normalized data is presented in the bottom panel. Bottom panel: Fits, the same as in Fig.1 but for the USA. The date 20th of March is indicated by a line in both panels. See also text.

It is interesting to compare the results for the three European countries to those obtained for the case of the USA presented in Fig.4. The latter are taken from another ongoing study by us and are probably a very good illustration of how difficult is to perform an analysis of the data, and moreover, formulate predictions over a long time range. One difficulty here is that the assumption for a more or less constant background turned out to be not completely justified (after about day 90). Therefore, we made the next simplest possible assumption for a linear dependence of a decreasing "background" till about day 100 which was subtracted from the normalized data in that time interval. For larger times, again a constant background was assumed and subtracted. After these operations, illustrated in the upper panel of Fig.4, the resulting background-subtracted data were analyzed as shown in the lower panel of Fig.4. This panel is otherwise very similar to what has been displayed in Figs.1,2 and 3 and especially regarding the complete set of fitted parameters (see Table 1). It is remarkable that the parameters in each column of that Table are not tremendously different with the exception of the ones which are related to the starting time of each wave and which provide an unique time-stamp for the developments in each country. A common feature which is also interesting is that with the exception of the UK, the date 20th of March indicated in all figures (Figs. 1, 2, 3 and 4), in all other cases considered is associated with a change in the development of the Pandemic revealed both by data and fits. This observation may be related to the beginning of Spring in these North-Hemisphere countries (with the Omicron wave in the UK starting only a bit earlier), adding another element to the complexity of treating the problem. It is not our intention to formulate rules for lifting restrictions, but a criterion on how much the data and fits are above the acceptable "background" at a given planned time may be useful. Thereby, mathematical modeling has still to be employed along other guiding lines of social, health and economics character, in general. On the other hand, it might be that with the newly emerging variants of SARS-CoV-2 and their sub-variants (e.g. Omicron's BA.4, BA.5) the very notion of immunity in the case the COVID-19 has to be made more flexible e.g. by taking into account the possibility for immune evasion^{18,19} with respect to both vaccines and acquiring immunity after recovery. Indications for smaller effects of that kind have been established already earlier (e.g. variable duration of protection), and therefore the question arises if there is way to provide support to immunity in advance to every new threat, and

how fast the health systems in so much strongly differing countries can react to such threats. For sure, it will be better than in early 2020, but still...

Conclusions

The COVID-19 Omicron waves in the UK, France and Germany till June 2022 are considered in retrospective within a new approach⁸ for modeling Epidemics whose formalism was improved in the present work. The improvement consists mainly in ensuring better conditions for continuity when solving the underlying differential equations for consecutive waves. In addition, a superposition of independent waves, related to different pathogens, is involved in the considerations. The daily observed new infection cases are described in a reasonable way after normalization. The position of the calculated first Omicron wave peaks in the Pandemic indicates a transition from the third to the fourth generation of infected people. The parameters derived by reproducing the data in the three countries considered are consistent with each other and with the preliminary ones derived for the USA (most important are the infection and recovery rates). The only exception is the starting time of each wave which provides an unique time-fingerprint for the developments in each country. A correlation is observed in all considered cases in Europe between initialization of relaxing restrictions and a new wave. However, in some cases nearly at the same time a new independent wave starts, related most probably to a different version of the pathogen. It is not possible to formulate fixed rules for lifting restrictions, but a criterion on how much the data and fits are above the acceptable "background" at a given planned time may be useful. Thereby, mathematical modeling should still to be employed along other guiding lines of social, health and economics character.

On the other hand, it might be that with the newly emerging variants of SARS-CoV-2 and their sub-variants the very definition of immunity in the case the COVID-19 has to be made more flexible. Apart its temporary character (with respect to both vaccines and acquiring after recovery) established already earlier, the question arises if there is a way to provide protection in due time against every new threat and how fast the health systems in so much strongly differing countries can react to such threats.

Finally, the demonstration that the problem of describing in a reasonable way Epidemics by using consecutive waves and superposition of waves caused by different pathogen variants opens the possibility to investigate the COVID-19 Pandemic in its full time range since early 2020 to present. It is our intention to work on that problem

in the future with the two-fold aim to obtain additional useful information on that particular Pandemic in some country (or region) and develop further the model (and software) toward readiness (preparedness) to meet next possible challenges when they come.

The present work represents only a mathematical modeling of the Pandemic and does not deal with any other aspect of health, social or economic character as well with throwing away any further possibility for unexpected developments.

Conflicts of Interest Statement

The author declares that no conflicts of interest have to be disclosed.

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