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RESEARCH ARTICLE

UNDERSTANDING THE DYNAMICS OF COVID-19 TRANSMISSION IN PAMPANGA PHILIPPINES: MODELING WITH A SYSTEM OF DIFFERENTIAL EQUATIONS

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ABSTRACT

This study introduced and solved a system of differential equations aimed at modeling the coronavirus disease 2019 (COVID-19) transmission dynamics in the province of Pampanga. Specifically, a Susceptible, Infected, Recovered, Deceased (SIRD) model was developed and built upon the foundational SIR model devised by Kermack and McKendrick in 1927.¹ Various methods were employed to solve the model. Initially, the analytical solution for the rate of change of infected individuals over time $\frac{dI}{dt}$ was determined. Subsequently, model parameters were identified through an optimization process using the Microsoft Excel Solver. The Runge-Kutta fourth order (RK4) method, implemented in Scilab 6.1.1, was utilized to approximate the numerical solution for the rates of change of susceptible $\frac{dS}{dt}$, recovered $\frac{dR}{dt}$, and deceased $\frac{dD}{dt}$ over time. The findings underscored the significance of several parameters—namely, the transmission rate (τ), removal rate (combining recovery (ρ) and deceased rate (φ)), the proportion of the infected population properly wearing face masks (μ), the proportion disinfecting regularly (δ), and the proportion practicing isolation or social distancing (ϵ)—in shaping the transmission dynamics of COVID-19 in Pampanga. The values of these model parameters reflect the effectiveness of governmental responses and actions in managing, controlling, and mitigating the spread of COVID-19, as well as the extent of public cooperation and compliance with COVID-19 directives and advisories.

Keywords: COVID-19, Differential Equations, Runge-Kutta fourth order (RK4) method, SIRD model

Introduction

In the 21st century, humanity has faced an unprecedented global challenge in the form of the COVID-19 pandemic. The novel coronavirus, severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), spread across continents, creating a complex patterns of transmission dynamics that have challenged the world. To comprehend and control the spread of the virus, scientists have used the tool of mathematical modeling, specifically employing systems of differential equations.

Traditional epidemiological models that are grounded in the principles of compartmental modeling have proven effective in understanding and predicting the course of infectious diseases.^{1,2} These models divide the population into different compartments, representing distinct stages of infection, and utilize differential equations to describe the flow of individuals between these compartments over time.^{1,2}

In the context of COVID-19, a typical compartmental model may include categories such as susceptible individuals, exposed (but not yet infectious) individuals, infectious individuals, and recovered or deceased individuals.^{2,3} By expressing the rates at which individuals move between these compartments as differential equations, researchers can simulate and analyze the spread of the virus within a population.³ The parameters of these equations are informed by biological characteristics of the virus, human behavior, and societal factors.

As we explore the principle of this modeling approach, it becomes clear that differential equations provide a framework to understand the evolving nature of the pandemic. By

integrating real-world data, researchers can refine their models to reflect the complexities of COVID-19 transmission, allowing for more accurate predictions and the identification of effective public health interventions.

This research study examined the complex dynamics of COVID-19 transmission in Pampanga, Philippines using mathematical modeling. We explored the fundamentals of compartmental differential equation-based models to deepen our understanding of the virus's spread and inform strategies to mitigate its impact. As we navigate through the complexities of this global health crisis, the integration of mathematical modeling becomes increasingly vital in the pursuit of managing and overcoming the challenges posed by COVID-19.

Accurate models are crucial for understanding the complexities of the COVID-19 outbreak and evaluating various mitigation strategies. Numerous studies have explored various modeling approaches to scrutinize the virus's transmission dynamics within populations.⁴⁻¹³ Additionally, some studies have conducted bibliographic reviews to trace the evolution of COVID-19 transmission dynamics research.¹⁴ Other studies have also undertaken systematic reviews of existing knowledge to create papers with potentially significant impacts.¹⁵

This current study extends previous research by employing an advanced Susceptible-Infected-Recovered-Deceased (SIRD) model to depict the dynamics of COVID-19 transmission in the province of Pampanga, Philippines. The SIRD model is derived from the SIR model initially formulated by Kermack and McKendrick in 1927.¹ Researchers have utilized the SIR model to establish a theoretical

framework for investigating virus transmission within a specific population, as it can offer insights and predictions regarding transmission dynamics beyond what recorded data alone can provide.¹⁶

Specifically, this paper builds on the previous study by Mallari and Mendoza,¹⁷ which used the SIRD model to explore the dynamics of the COVID-19 transmission in Pampanga from March 2020 to December 2021. The study found that the five model parameters used – transmission rate (τ), recovery (λ) and deceased rate (ω), proportion of the population using face mask (θ) and the efficacy of face mask (ϵ) – were significant factors driving the rise of COVID-19 cases. To further explore the dynamics of virus transmission as well as factors that affect it, this paper introduces three new parameters in the SIRD model. The newly defined parameters are the proportion of the infected population disinfecting regularly (δ), proportion of the infected population who were wearing face mask properly (μ), and proportion of the infected population practicing isolation or social distancing (ϵ).

Methodology

A. Data Source

COVID-19 cases spanning from March 2020 to October 2021 were gathered from the Pampanga Information Office (PIO). The dataset was segmented into four distinct time periods for analysis.

B. Model Development, Formulation and Solution

In this research, the removed compartment in Kermack and McKendrick's well-known compartmental SIR model from 1927 was divided into recovered and deceased compartments,

resulting in an extended SIRD model. This modification was undertaken to accurately described the recovered and deceased cases of COVID-19 in the province of Pampanga.

Consequently, this study made a mathematical description of the compartments composing the SIRD model with the following system of differential equations:

$$\frac{dS}{dt} = -\tau(1 - \mu\epsilon\delta)S \frac{I}{N} \quad (1)$$

$$\frac{dI}{dt} = \tau(1 - \mu\epsilon\delta)S \frac{I}{N} - (\rho + \varphi)I \quad (2)$$

$$\frac{dR}{dt} = \rho I \quad (3)$$

$$\frac{dD}{dt} = \varphi I \quad (4)$$

where:

- τ - Transmission rate (risk of infection per contact)
- ρ - recovery rate
- φ - deceased rate
- μ - proportion of the infected population who were wearing facemask properly
- ϵ - proportion of the infected population practicing social distancing
- δ - proportion of the infected population constantly disinfecting

The system of differential equations (1), (2), (3), and (4) characterizing the dynamics of COVID-19 transmission was solved using a combination of analytical and numerical methods. The function representing the number of infected individuals at a given time (t), denoted by $I(t)$, was derived by solving equation (2) analytically. This was based on the assumption

that the susceptible (S) remains approximately equal to the population (N) throughout the time span, i.e., $S/N \approx 1$. Consequently, equation (2) could be simplified and solved analytically through the separation of variables.

The particular solution for solving equation (2) is presented below.

$$I(t) = I_0 e^{(\tau(1-\mu\epsilon\delta) - (\rho+\varphi))t} \quad (5)$$

Where, I_0 represents the initial infected cases at time $t = 0$. The unknown model parameters $\tau, \mu, \epsilon, \delta, \rho, \varphi$ in equation (5) were deduced by fitting the observed infected cases through the least square criterion using optimization in Microsoft Excel Solver.

To address the challenge of solving the analytical solutions for the rates of change in the SIRD model—specifically $\frac{dS}{dt}$ (susceptible), $\frac{dR}{dt}$ (recovered), and $\frac{dD}{dt}$ (deceased)—a numerical method was utilized. Specifically, the fourth-order Runge-Kutta (RK4) method was employed to approximate the solutions for equations (1), (3), and (4) of the SIRD model.

Results

Upon obtaining equation (5) as the particular solution of equation (2) through analytical methods, the model parameters $\tau, \mu, \epsilon, \delta, \rho$ and φ were derived using Microsoft Excel Solver. The corresponding values of these parameters are detailed in Table 1 below.

Table 1. Model parameters of the solution function $I(t)$ in four periods, obtained through optimization

Model parameters	Initial period	2 nd Period	3 rd Period	4 th Period
Risk of infection per contact (τ)	0.396496123	0.226924	0.171810134	0.153547136
Recovery rate (ρ)	0.0753197	0.0647	0.06693498	0.0716
Deceased rate (φ)	0.0186	0.00356	0.003507102	0.003619
Proportion of the infected population who were wearing facemask properly (μ)	0.4884	0.53466	0.61487768	0.52641966
Proportion of the infected population practicing social distancing (ϵ)	0.4884	0.53466	0.61487768	0.52641966
Proportion of the infected population constantly disinfecting (δ)	0.4884	0.53466	0.61487768	0.52641966

With these acquired model parameters, the study assess the value of the solution function $I(t)$ at any given time t .

To obtain the approximate solution function $\hat{I}(t)$ using the Runge-Kutta Method, consider the initial value problem given by:

$$\frac{dI}{dt} = \tau(1 - \mu\epsilon\delta)S \frac{I}{N} - (\rho + \varphi)I = f(t, I)$$

with $I_0 = 3$ when $t = 0$.

Define $\Delta t > 0$ and proceed with the following steps:

1. $k_1 = f(t_i, y_i)$
2. $k_2 = f\left(t_i + \frac{\Delta t}{2}, I_i + \Delta t \frac{k_1}{2}\right)$
3. $k_3 = f\left(t_i + \frac{\Delta t}{2}, I_i + \Delta t \frac{k_2}{2}\right)$
4. $k_4 = f(t_i + \Delta t, I_i + \Delta t k_3)$

Then, update I and t using:

$$I_{i+1} = I_i + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$t_{i+1} = t_i + \Delta t,$$

This process is iterated for $i = 0, 1, 2, 3, \dots, n$. The Runge-Kutta method calculates approximate values $I_{t_1}, I_{t_2}, \dots, I_{t_n}$. By implementing the Runge-Kutta algorithm in Scilab 6.1.1, we generate graphs that depict the approximate solution functions for $\hat{I}(t)$. These graphs are illustrated in Figure 1 below. Additionally, Figure 1 displays the graph representing the observed infected cases $I(t)$.

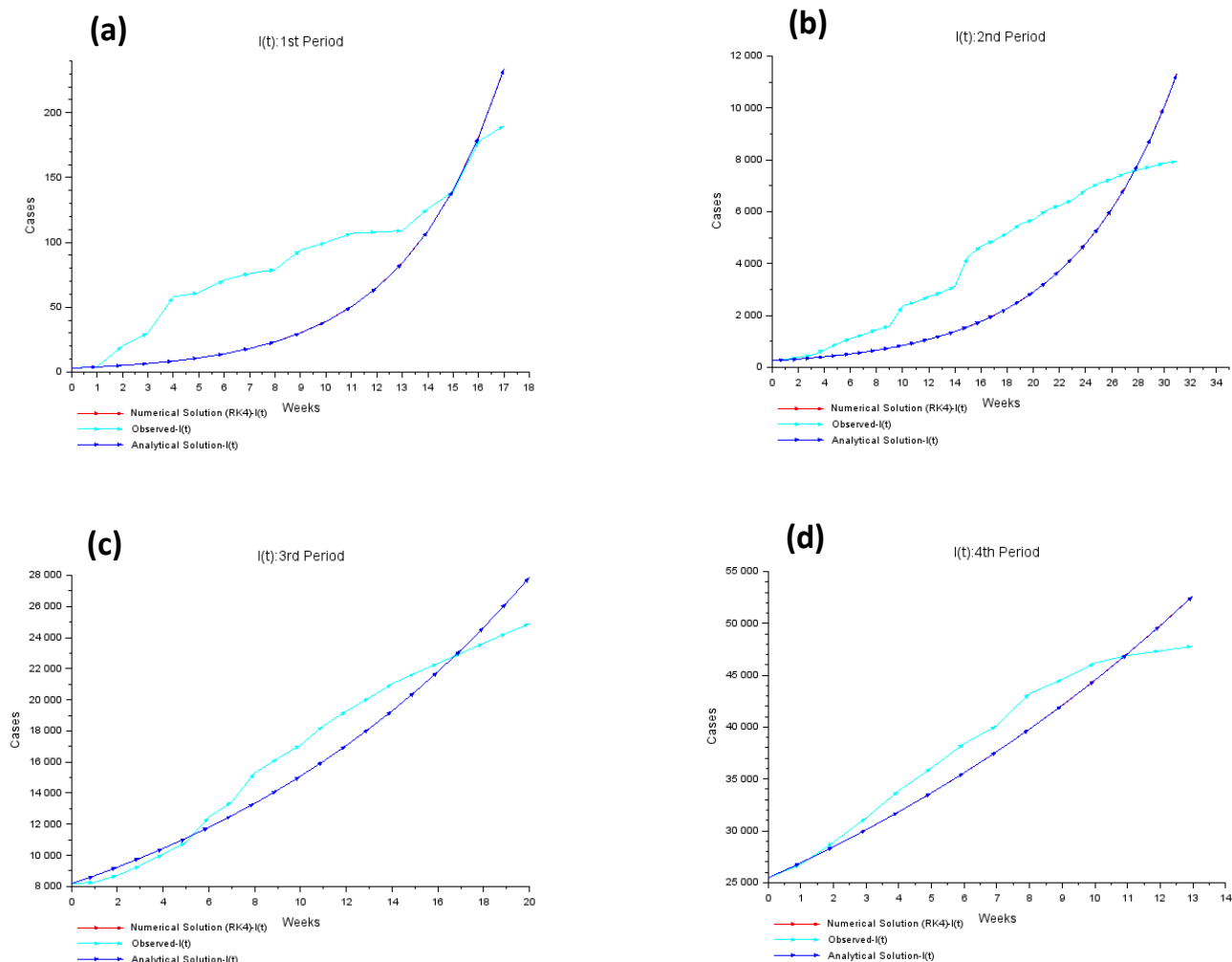


Figure 1. Solution functions of infected cases $I(t)$ over four distinct periods of COVID-19 cases in Pampanga, Philippines, spanning from March 2020 to October 2021.

Employing the RK4 method and utilizing the derived parameters in $I(t)$, the approximate solution functions for susceptible individuals $S(t)$, recovered individuals $R(t)$, and deceased $D(t)$ were estimated. This was achieved by

executing an RK4 program code written in Scilab 6.1.1.

The figures below show the graphs of the modeled and observed susceptible individuals, recovered individuals, and deceased cases.

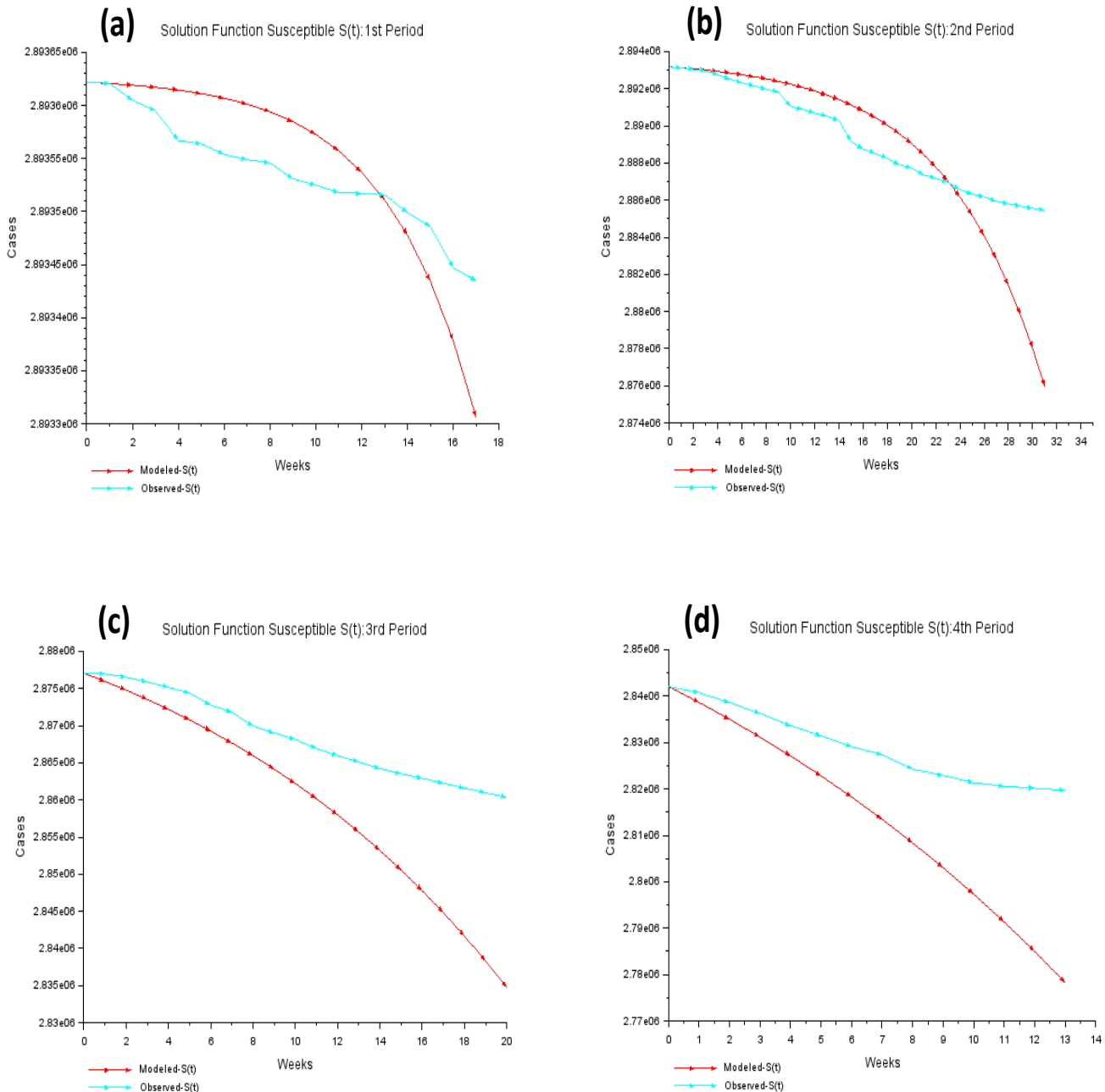


Figure 2. Solution functions of Susceptible cases $S(t)$ over four distinct periods of COVID-19 cases in Pampanga, Philippines, spanning from March 2020 to October 2021.

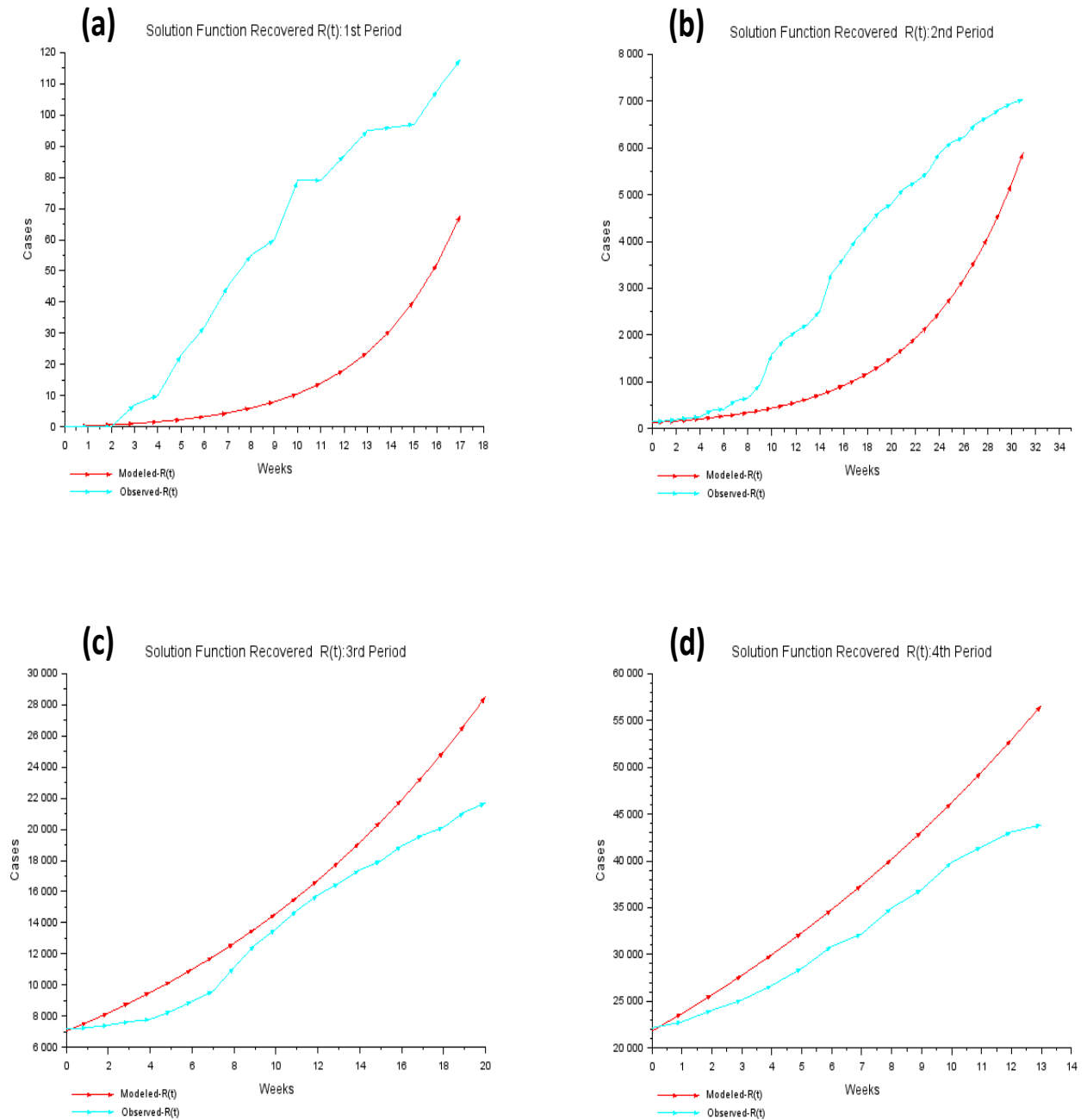


Figure 3. Solution functions of Recovered cases $R(t)$ over four distinct periods of COVID-19 cases in Pampanga, Philippines, spanning from March 2020 to October 2021.

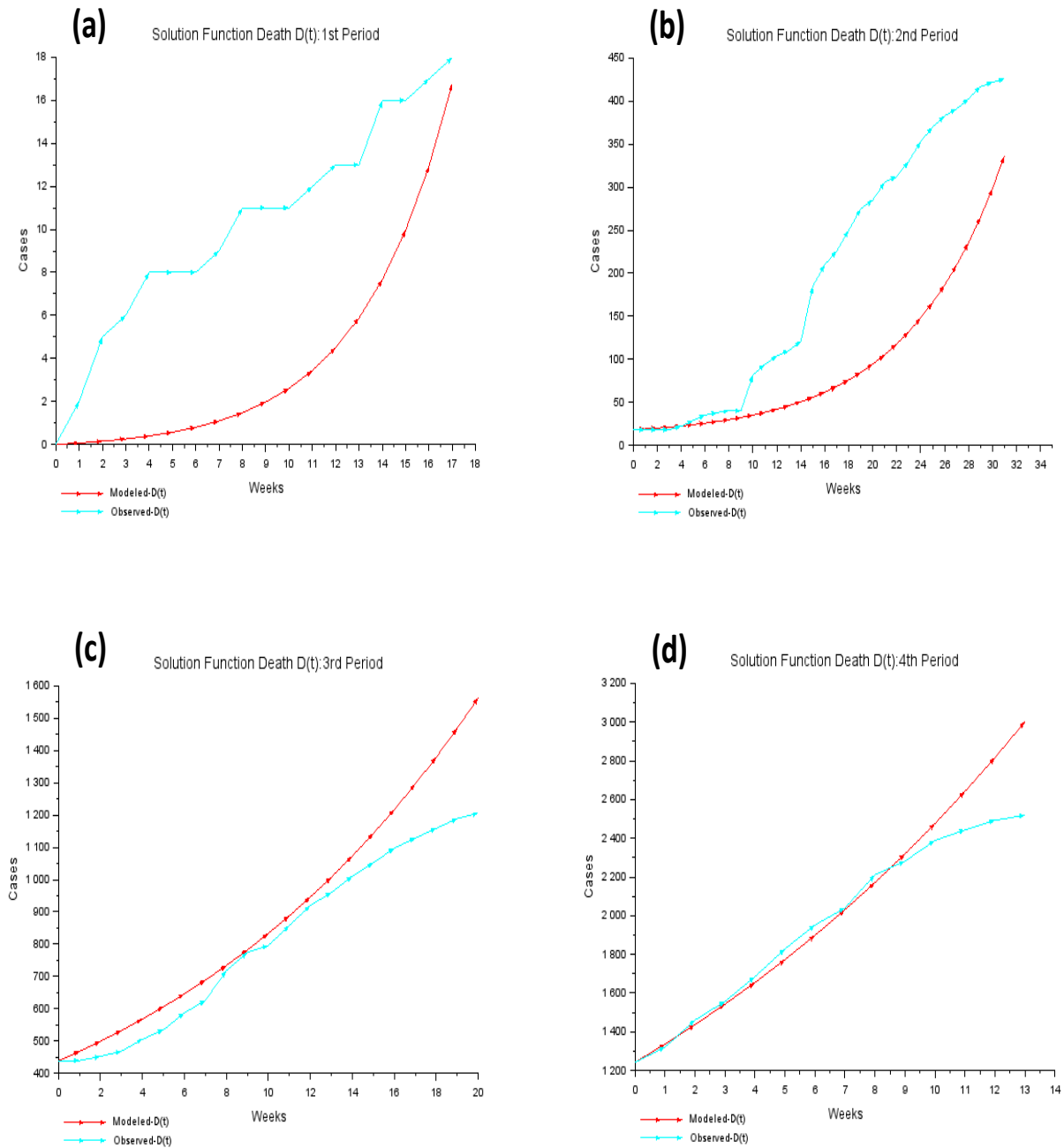


Figure 4. Solution functions of Deceased cases $D(t)$ over four distinct periods of COVID-19 cases in Pampanga, Philippines, spanning from March 2020 to October 2021.

The gathered data from PIO, as well as solved and derived data are summarized in Table 2 below.

Table 2: *Initial Values and Model parameters for the COVID-19 SIRD Model in Magalang, Pampanga*

Parameter	Value	Source
Population (N)	2,893,625	Given
Initial time (t₀)	March 7, 2020 (day 67)	Given
Initial susceptible (S₀)	$S_0 \approx N$	Given
Initial infected (I₀)	3	Given
Initial recovered (R₀)	0	Given
Initial deceased (D₀)	0	Given
Incubation period (infected but not yet infectious)	5.5 days	Source
Risk of infection per contact (τ)	0.1-0.4	Solved
Recovery rate (ρ)	0.06-0.08	Solved
Deceased rate (φ)	0.0035-0.0187	Solved
Time Progression of recovery	(12-15) days	Solved
$\frac{\text{Total recovered}}{\text{Total infected}}$	0.95	Solved
Time progression of deceased period	(54-281)days	Solved
$\frac{\text{Total deceased}}{\text{Total infected}}$	0.05	Solved
Removal rate($\rho + \varphi$)	0.068-0.094	Solved
Proportion of the infected population who were wearing facemask properly (μ)	49%-62%	Solved
Proportion of the infected population who practice social distancing (ϵ)	49%-62%	Solved
Proportion of the infected population who constantly disinfect (δ)	49%-62%	Solved
Basic reproduction number (R₀)	1.74-3.73	Solved

Discussions

The numerical and analytical solutions fitted with each other across all four periods, as illustrated in Figure 1. Additionally, both the numerical and analytical solutions demonstrate that the data points for $I(t)$ correlate with the observed data of infected cases for the specified four periods.

The transmission rate or risk of infection per contact (τ), the recovery rate (ρ), and the deceased rate (φ) were obtained through optimization. Table 2 presents the values of (τ), (ρ), and (φ) for the four periods, ranging from 0.15 to 0.4, 0.067 to 0.075, and 0.0035 to 0.0187, respectively.

Based on the obtained model parameters, this study was able to provide the model parameters used in solving the rate of change of susceptible with respect to time $\frac{dS}{dt}$, the rate of change of recovered with respect to time $\frac{dR}{dt}$, and rate of change of deceased with respect to time $\frac{dD}{dt}$ and eventually solved them numerically. Additionally, the model effectively depicted the behaviors of susceptible, recovered, and deceased individuals, and has a good fit with the actual observed data for these three compartments throughout the four periods, as depicted in Figures 2, 3, and 4.

All the model parameters mentioned above, each with its distinct value, influenced the upward trend in the infected, recovered, and deceased compartments, as well as the downward trend in the susceptible compartment across the four periods, as illustrated in Figures 1, 2, 3, and 4, respectively.

Moreover, the results of this study indicate that 48.8% to 61.5% of the infected population

adheres to minimum health protocols, including proper facemask usage, constant disinfection, and adherence to isolation or social distancing practices. Consequently, 38.5%-51.2% of the infected population does not strictly adhere to these essential health guidelines. These values of the model parameters demonstrated the effectiveness of the government's management and control program throughout the study period of the COVID-19 pandemic, as well as the public's response to the disease outbreak.

The study by Caldwell et al.⁷ emphasized the significant role of adherence to minimum health standards (MHS) in mitigating the initial COVID-19 outbreak in the Philippines. Consequently, the findings of this current study align with their observations.

Furthermore, the findings of this study can serve as a guideline for managing, mitigating, and controlling similar novel epidemiological disease outbreaks in future scenarios.

Meanwhile, the recovery period was identified to span 12-15 days, while the deceased period ranged from 54-281 days. Combining the recovery period and the deceased period with the fast-increasing trend of the infected compartment on a weekly basis, health facilities are strained and overwhelmed.

The percentage of individuals who fully recovered relative to the total number of infected cases was at 95%, while the proportion of total fatalities in relation to the total number of infected cases was determined to be 5%.

The removal rate ($\rho + \varphi$) was determined to be in the range of 0.068-0.094. Consequently, the days required for removal ($\frac{1}{(\rho + \varphi)}$) were calculated to be 12-15 days. Additionally, the

basic reproduction number, denoted as R_0 , was determined across all four periods and varied from 1.74 to 3.73.

Model Limitation

The developed SIRD model based on the historical data was designed for a brief period of COVID-19 transmission dynamics.

Considering the available observed data, the performance of the developed SIRD model could exhibit variability due to factors not explicitly considered in the analysis. These factors include government management and control program data related to mass testing, contact tracing, quarantine procedures, vaccination programs and other aspects which are not accounted for in this current study.

Conclusion

The analytical solution for the infected cases fitted the observed cases and able to make an analysis in the dynamics of COVID-19 transmission in the province of Pampanga. Utilizing the obtained model parameters through optimization we were able to provide the model parameters used in finding the functions susceptible $S(t)$, recovered $R(t)$ and deceased $D(t)$ for any time (t). Thus, the numerical solutions for the three compartments had a good fit with the observed data.

The developed model yielded essential parameters through optimization, including the transmission rate (τ), the removal rate (comprising recovery denoted by ρ and deceased rate denoted by φ), the proportion of the infected population wearing face masks properly (μ), the proportion disinfecting regularly (δ), and the proportion practicing

isolation or social distancing (ϵ). These parameters were significant contributors to the transmission dynamics of COVID-19 in Pampanga. Consequently, the study suggests that effective control, mitigation, and management of COVID-19 transmission hinge on the coordinated efforts of authorities and the public. Adherence to minimum health standards, such as wearing face masks properly, regular disinfection, and practicing social distancing, can substantially mitigate, if not entirely eliminate, the rapid spread of COVID-19 over a short period of time.

Recommendations

In future studies, it may be valuable to develop a mathematical model that encompasses additional parameters to account for factors such as mass testing, contact tracing, quarantine procedures, health facility utilization, vaccination, and other factors not presently addressed in the current study. These considerations can enhance the comprehensiveness of the model and better capture the various influences on the dynamics of COVID-19 transmission.

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None

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