A brief review of dynamic instability of a beam/plate in the magnetic field

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ABSTRACT

In this review, the fundamental results concerning the dynamic instability of a beam/plate structure in the magnetic field have been presented, that have been made over the last two decades, many of which are related to the mechanical model and responses of the magneto-elastic system. The review shows the basis equation of motion of a beam/plate in an oscillating magnetic field and touches on the dynamic instability behavior of a beam/plate system which is the isotropic material and composites made of the piezoelectric/ piezomagnetic materials.

1. Introduction

The dynamic instability of magneto-elastic structures has been one of practical interest in engineering. One of the complicated problems of magneto-solids mechanics is the treatment of the interaction of electromagnetic fields with deformable structures. Because of the particular magneto-solids mechanics arisen from nonlinear interactions between the nonlinear magnetization of structures and an applied magnetic field, dynamic instability of a beam/plate phenomenon has always attracted much attention in both theoretical and engineering science.

The practical significance of the magnetoelastic interactions has led to a large number of fundamental studies. Moon and Pao [1,2] proposed the first mathematical model with distributed magnetic forces/torques and experimentally discovered the magnetoelastic buckling and parametric instability of a cantilever beam-plate in a uniform transverse magnetic field. Based on the magnetic force on the distributed diploes and the theory of thin plates, Moon and Pao obtained the analytical expression of the critical magnetic field which has propelled many investigations. Afterwards, Dalrymple et al. [3,4], Miya et al. [5,6], Peach et al. [7], Christopherson et al. [8] etc. devoted their attention to finding an explanation for this result. Many investigators concentrated mainly on the effect of edge on the critical field, studied magnetoelastic buckling problem experimentally, and compared their experiment results to the Moon-Pao model [3-8]. Based on the Cauchy's equation of motion, Maxwell's equations and corresponding jump conditions, the theory of thin elastic plates subjected to a magnetic field and the dynamic equations of linear elastic solids, carrying large static, bias electromagnetic fields have been presented by Eringen [9].

Recently, many researchers have paid their attention to the mechanics problems as more magneto-electro-elastic (MEE) and magnetorheological elastomer (MRE) materials were used in technological applications. For example, more recent advances are the smart or intelligent materials where piezoelectric and/or piezomagnetic materials are involved [10-17]. These materials have the ability of converting energy from one form to the other (among magnetic, electric, and mechanical energies) and make them suitable for application in sensing and actuating devices, vibrations control, energy harvesting and smart structure technology [16,17]. Furthermore, composites made of piezoelectric/piezomagnetic materials exhibit a magneto-electric effect that is not present in single-phase piezoelectric or piezomagnetic materials.

Over the past investigations on the dynamic problem of magnetic structural elements, many solutions have been

devoted to this particular magneto-solids mechanics due to its many engineering applications. Here, the paper focuses on the recent magneto-elastic dynamic instability accomplishments of a beam/plate in a magnetic field, especially within the last two decades, but early, pioneering papers and the piezomagnetic materials works are also emphasized.

2. The basis equation of motion

If the beam or plate is in an oscillating magnetic field, it may exhibit parametrically excited oscillations. The effect of an external magnetic field on an elastic body can be divided into parts, one arises from the conduction and the other is caused by the magnetization in the body [2]. The equation of motion is

$$D\frac{\partial^4 u}{\partial x^4} + b\frac{\partial^2 u}{\partial x^2} = -2\rho h \frac{\partial^2 u}{\partial t^2} \qquad (1)$$

where $D = \dfrac{2h^3 E}{3(1-v^2)}$ is the flexural rigidity, E is the Young's modulus, v is the Poisson's ratio and $2h$ is the thickness of the plate. Based on the sinusoidally deformed plate of infinite extent with wavelength $\lambda (= 2\pi/k)$, the proportional b was found as

$$b = 2\chi^2 B_0^2 \sinh kh / \mu_0 \mu_r k\Delta,$$

and when $kh << 1$ and $\chi >> 1$,

$$b \cong 2\chi^2 B_0^2 h / \mu_0(1+\mu_r kh) \qquad (2)$$

where B_0 is the transverse magnetic field, $\chi = 1 - \mu_r$ is the susceptibility, μ_0 is the permeability of the vacuum, μ_r is the relative permeability, k is the wave number, and $\Delta = \mu_r \sinh kh + \cosh kh$ [2]. The displacement u(x,t) is assumed as

$$u(x,t) = [A_1 \sin(kx + A_2) + A_3] f(t) \qquad (3)$$

which is a solution of (1) if $A_3 = 0$ and $f(t)$ satisfies the following equation

$$\frac{d^2 f}{dt^2} + \frac{Dk^4}{2\rho h}(1 - \frac{b}{Dk^2})f = 0 \qquad (4)$$

Eqs. (1)-(4) form the basis of the dynamics of a beam-plate in a transverse magnetic field. When the applied field is assumed to be uniform but time-dependent $B_0 = B_0 \cos \Omega t$, and b is treated as a function of time. The basic equation of motion of a simply supported beam-plate in an alternating magnetic field can be obtained and shown as following [2]:

$$\frac{d^2 f}{dt^2} + \omega_L^2 (1 - 2\eta \cos 2\varpi t)w = 0 \qquad (5)$$

where $\omega_L^2 = \omega_0^2(1 - B_r^2/B_c^2) = \omega_0^2(1 - \bar{B}^2)$,

$2\eta = B_m^2/(2B_c^2 - B_0^2) = \bar{B}^2/(1 - \bar{B}^2)$,

$\bar{B}^2 = B_0^2/2B_c^2 = B_r^2/B_c^2$,

$\omega_0^2 = k^4 (D/2\rho h) = (n\pi/L)^4 (D/2\rho h)$, and

$$B_c^2 = \frac{\mu_0 \mu_r E}{3\chi^2 (1-v^2)} (\frac{n\pi h}{L})^2 (1 + \mu_r \frac{n\pi h}{L})$$

Therefore, the regions of linear stability and instability for parametric resonances in an oscillating magnetic field were obtained by solving the well-known Mathieu equation. Moon and Pao [1, 2] gave us a better understanding of the magneto-elastic buckling phenomenon, but left the interesting issue of how to explain the big discrepancy between the theoretical predictions and the experimental data for the critical magnetic field. Dalrymple et al. [3,4], Peach et al. [7], and Christopherson et al. [8] studied the buckling problems experimentally and compared their results with Moon [2]. Miya et al. [5,6] applied experimental and finite element methods to study magnetoelastic buckling on a cantilevered beam-plate and their results had been made to explain well the broad discrepancy between theoretical and experimental values of the magneto-elastic buckling field of a ferromagnetic beam plate. Eringen [9] derived the fields equations for elastic plates subjected to small dynamical loads and electromagnetic fields.

3. The dynamic instability of a beam/plate in the magnetic field

It is well known that motion of an electrically conducting structure in a magnetic field can induce an electric current which interacts with the magnetic field. This magnetic damping is known to have a destabilizing effect when the structure is subjected to an in-plane longitudinal force. In this section, the dynamic behaviors of a beam/plate system are presented by the isotropic material and the composites made of piezoelectric /piezomagnetic materials.

3.1. The isotropic material of the beam/plate

In the past, several researches concerned active reduction of vibration amplitudes and the destabilizing effect due to the magnetic damping. The differential equation for the motion of the plate strip in a magnetic field parallel to the plan of motion was first derived by Lee [18] with the destabilizing effect due to the magnetic damping. Lu et al. [19] derived a mechanical model of a magnetoelastic buckled beam subjected to an external axial periodic force in a periodic transversal magnetic field. Using the Hamilton's principle, Shih et al. [20] and Wu et al. [21] derived the equation of motion for elastic beam under pulsating axial load and oscillating transverse magnetic field. The equation was reduced to a well-known Mathieu's equation and dynamic stability of the system for various loading conditions had been discussed.

Bae et al. [22] presented a modeling

technique for the effective eddy current damper and vibration suppression of a cantilever beam using the eddy current damper. For the eddy current damper, it was derived using the electromagnetic theory combined with the image method. Moreover, the parametric instability of a beam under electromagnetic excitation was investigated experimentally and analytically by Chen and Yeh [23]. The instability regions of the system were found to be the functions of the modal parameters of the beam and the position, the stiffness of the electromagnetic device for various cantilevered beams with the assumed-modes method. In addition, the non-linear behavior of a single-link flexible viscoelastic Cartesian manipulator and the instability regions of a cantilever beam with tip mass subjected to time-varying axial loading and magnetic field had been studied by Pratiher and Dwivedy [24-26]. They concluded that with increase in damping and static axial load or decrease in mass ratio the instability region decreases and one may control the vibration of a system using the required magnetic field.

From the above investigations, the dynamic instability of the beam in a periodic transversal magnetic field can be summarized to be the functions of the modal parameters of the beam, the stiffness of the electromagnetic device, and the effects of the applied alternating magnetic field, damping and induced currents.

Based on the von Karman model of plates, Zhou et al. [27-29] presented the buckling and post-buckling behavior of a soft ferromagnetic beam-plate by taking into account magnetoelastic interaction and non-linear effect of large deflections. Zhou et al. showed that the predictions from their model were much more close to the experimental data for ferromagnetic plates buckling in a transverse field. Based on a linearized magnetoelastic theory and perturbation method, Zheng and Wang [30] developed a theoretical model to describe a magnetic force arisen from an interaction between the nonlinear magnetization of structures made of soft ferromagnetic materials and an applied magnetic field which showed that the magnetic damping extends the stable regions of parametric excitation to a certain extent.

Baghdasaryan et al [31] formulated and investigated the problem of vibrations of conductive plate in a longitudinal magnetic field with the basis of Kirchhoff hypothesis. The results [30] showed that: (1) in the case of perfectly conductive plates, the magnetic field constricts essentially the width of the main area of dynamic instability; (2) if the plate's material has the finite electro conductivity, then the certain value of the intensity of external magnetic field exists, the exceeding of which excludes the possibility of appearance of parametric type resonance.

In all the above investigations, the

authors studied the magneto-solids mechanics arisen from nonlinear interactions between the nonlinear magnetization of structures and an applied magnetic field. The effects of variation of the thermal loads and deformations on the dynamic instability of the structures were not considered synchronously. By means of the magnetic field perturbation technique and the finite element method, Wang et al. [32] investigated the effects of thermal and magnetic fields on the magneto-thermo-elastic bending and buckling. Wu [33,34] developed a theoretical model for a pinned beam subjected to an alternating magnetic field and thermal load with the linear/nonlinear strain and a physically nonlinear constitutive law. The dynamic instabilities and transient vibrations of a bimaterial beam with alternating magnetic fields and thermal loads had also been investigated by Wu [35] which indicated that the responses of the dynamic instability were influenced by the temperature increase, the magnetic field, the thickness ratio, the excitation frequency, and the dimensionless damping ratio.

3.2. The composites made of piezoelectric/piezomagnetic materials

Recently, advances have been made in smart or intelligent materials where piezoelectric and/or piezomagnetic materials are involved. These composite materials exhibit a desirable coupling effect between electric and magnetic fields, which are useful in intelligent structure applications. Pan and Heyliger [10,11] presented an exact closed-formsolution for the static deformation of the four cases of the sandwich piezoelectric/magnetostrictive plate. Pan and Heyliger [11] observed the features: (1) Some of the modes are purely elastic, (2) Piezoelectric/magnetostrictive coupling does not produce electric and magnetic potentials, and the elastic displacements are nearly identical in both the purely elastic and piezoelectric/ magnetostrictive cases, (3) For the piezoelectric/magnetostrictive coupling mode, all the mode shapes, especially the elastic displacement component in the *z*-direction and electric and magnetic potentials depend strongly upon the material properties and stacking sequences.

Jia et al [36] analyzed firstly the properties of giant magnetostrictive thin films (GMFs) at low magnetic fields, including the large magnetostriction, the soft magnetization and the hysteresis under internal stress (prestress). Anandkumar et al. [37] studied the free vibration studies of multiphase and layered magneto-electro-elastic beam for different boundary conditions. The results conclude that multiphase material properties vary and are dependent on the ratio of fiber material to matrix material for BaTiO3–CoFe2O4 composite, the volume fraction of BaTiO3

is increased in steps of 20% to obtain different multiphase beam configurations.

Furthermore, magnetorheological elastomer (MRE) has been implemented to achieve controllable properties in sandwich structures with MRE embedded cores between elastic or metal faces. The magnetorheological elastomer (MRE) is embedded in the viscoelastic core to actively attenuate vibration in sandwich structure by applying a suitable magnetic field. Free and forced vibration of sandwich beams with viscoelastic damping cores was carried out by Zhou and Wang [38-40]. Nayak et al. [41-43] studied the dynamic analysis of a three-layered symmetric sandwich beam with magnetorheological elastomer (MRE) embedded viscoelastic core and conductive skins subjected to a periodic axial load under various boundary conditions. The results showed that MRE partly works in shear mode and hence the dynamic properties of the sandwich beam can be controlled by magnetic fields due to the field dependent shear modulus of MRE material.

For the MRE system parameters considered, it has been observed that with increase in magnetic field strength, length of MRE patch and the core thickness, the instability region decreases. With the increase in percentage of iron particles in case of MRE patch containing iron particles, the instability regions decreases [43]. Moreover, the system becomes unstable for a higher value of amplitude of dynamic loading.

Actually, recent modeling works on the active reduction of vibration amplitudes and the destabilizing effect in a magnetic field have been developed for various structure elements. For example, Kong et al. [44] presented the exact expressions of thermo-magneto-stress responses and perturbation response of magnetic field vector in a conducting non-homogeneous hollow cylinder. Wang et al. [45] investigated the dynamics and instability of current-carrying slender microbeams immersed in a longitudinal magnetic field by considering the material length scale effect of the micro beam. Free transverse vibration and instability of current-carrying nanowires immersed in a longitudinal magnetic field are of concern. Kiani [46, 47] developed a mathematical model to explain free transverse vibrations of very long current-carrying nanowires subjected to longitudinal magnetic fields. The analytical expressions of dynamic transverse displacements as well as natural frequencies of the magnetically affected nanowire for carrying electric current were obtained and analyzed.

4. Conslusions

In the past 20 years, many solutions are devoted to this particular magneto-solids mechanics through the development and validation of advanced theoretical

models. The treatment of the interaction of electromagnetic fields with deformable structures, the dissipative or damping capacities of the system, and the destabilizing effect, are now well known. The basic theoretical models of the magneto-solids system have now reached maturity and are being widely used for various structure elements.

In this brief review, the basis equation of motion of a beam in an oscillating magnetic field and the dynamic instability behavior of a beam/plate system are presented. In mechanical systems, the instability behavior of systems depends upon the boundary conditions, the mode of excitation, various material properties and the dissipative or damping capacities of the system itself. Meanwhile, the dynamic properties of the sandwich beam can be controlled by magnetic fields due to the field dependent shear modulus of MRE material. However, with development of new MREs, the variation of core loss factor and shear modulus with magnetic field may have a different characteristic which will lead to some other result. As with magneto-solids mechanics, the development of thorough theoretical models and their systematic validation against experiments will provide the necessary understanding for the nonlinear magnetization of structures.

References

[1] F.C. Moon, Y.H. Pao, Magnetoelastic buckling of a thin plate, J. Appl. Mech. 35 (1968) 53–58.

[2] F.C. Moon, Y.H. Pao, Vibration and dynamic instability of a beam-plate in a transverse magnetic field, J. Appl. Mech. 35 (1969) 92–100.

[3] J.M. Dalrymple, M.O. Peach, G.L. Viegelahn, Magnetoelastic buckling : Theory versus experiment. Exp. Mech. 16 (1976) 26–31.

[4] J.M. Dalrymple, M.O. Peach, G.L. Viegelahn, Edge effect influence on magnetoelastic buckling of rectangular plates, J. Appl. Mech. 44 (1977) 305–310.

[5] K. Miya, K. Hara, Y. Tabata, Experimental and theoretical study on magnetoelastic of a ferromagnetic cantilevered beam-plate, J. Appl. Mech. 45 (1978) 355–360.

[6] K. Miya, T. Takagi, Y. Ando, Finite element analysis of magnetoelastic buckling of a ferromagnetic beam-plate, J. Appl. Mech. 47 (1980) 377–382.

[7] M.O. Peach, N.S. Christopherson, J.M. Dalrymple, G.L. Viegelahn., Magnetoelastic buckling: why theory and experiment disagree. Exp. Mech. 28 (1988) 65–69.

[8] N.S. Christopherson, M.O. Peach, J.M. Dalrymple, Magnetoelastic deformation Experiment: Bending of a thin plate. Exp. Mech. 29 (1989) 432–436.

[9] A. C. Eringen, Theory of electromagnetic elastic plates. Int. J. Eng. Sci. 27 (1989) 363–375.

[10] E. Pan, Exact solution for simply supported and multilayered magneto-electro-elastic plates, J. Appl. Mech. 68 (2001) 608–618.

[11] E. Pan, P.R. Heyliger, Free vibration of simply supported and multilayered magneto-electro-elastic plates, J. Sound Vib. 252 (2002) 429–442.

[12] A.B. Temnykh, R.V.E. Lovelace, Electro-mechanical resonant magnetic field sensor, Nu. In. Meth. Phy. Res. A 484 (2002) 95–101.

[13] J. Chen, H. Chen, E. Pan, P.R. Heyliger, Modal analysis of magneto-electro-elastic plates using the state-vector approach, J. Sound Vib. 304 (2007) 722–734

[14] J. Chen, E. Pan, H. Chen, Wave propagation in magneto-electro-elastic multilayered plates, In. J. Solids. Stru. 44 (2007) 1073–1085.

[15] A. Milazzo, A one-dimensional model for dynamic analysis of generally layered magneto-electro-elastic beams, J. Sound Vib. 332 (2013) 465–483.

[16] S. Priya, R. Islam, S. Dong, D. Viehland, Recent advancements in magnetoelectric particulate and laminate composites, J. Elec. 19 (2007) 149–166.

[17] C.W. Nan, M. Bichurin, S. Dong, D. Viehland, G. Srinivasan, Multi ferroic magnetoelectric composites: historical perspective, status, and future directions, J. of Appl. Phys. 103 (2008) 031101.

[18] J.S. Lee, Destabilizing effect of magnetic damping in plate strip, J. Eng. Mech. 118 (1992) 161–173.

[19] Q.S. Lu, C.W.S. To, K.L. Huang, Dynamic stability and bifurcation an alternating load and magnetic field excited magnetoelastic beam, J. Sound Vib. 181 (1995) 873–891.

[20] Y.S. Shih, G.Y. Wu, J.S. Chen, Transient vibrations of a simply-supported beam with axial loads and transverse magnetic fields, Mech. Struc. Mach. 26 (1998) 115–130.

[21] G.Y. Wu, R. Tsai, Y.S. Shih, The analysis of dynamic stability and vibration motions of a cantilever beam with axial loads and transverse magnetic fields, J. Acoust. Soc. of ROC 4 (1997) 40–55.

[22] J.S. Bae, M.K. Kwak, D.J. Inman, Vibration suppression of a cantilever beam using eddy current damper, J. Sound Vib. 284 (2005) 805–824.

[23] C.C. Chen, M.K. Yeh, Parameter instability of a beam under electromagnetic excitation, J. Sound Vib. 240 (2001) 747-764.

[24] B. Pratiher, S.K. Dwivedy, Parametric instability of a cantilever beam with magnetic field and periodic axial load, J. Sound Vib. 305 (2007) 904-917

[25] B. Pratiher, S.K. Dwivedy, Non-linear vibration of a single link viscoelastic Cartesian manipulator, In. J. Non. Mech. 43 (2008) 683-696.

[26] B. Pratiher, S.K. Dwivedy, Non-linear dynamics of a soft magneto-elastic Cartesian manipulator, In. J. Non. Mech. 44 (2009) 757-768.

[27] Y.H. Zhou, X.J. Zheng, A general expression of magnetic force for soft ferromagnetic plates in complex magnetic fields, In. J. Eng. Sci. 35 (1997) 1405–1407.

[28] Y.H. Zhou, K. Miya, A theoretical prediction of natural frequency of a ferromagnetic beam-plate with low susceptibility in an in-plane magnetic field, J. of Appl. Mech. 65 (1998) 121–126.

[29] Y.H. Zhou, Y.W. Gao, X.J. Zheng, Q. Jiang, Buckling and post-buckling of a ferromagnetic beam-plate induced by magnetoelastic interactions, Int. J. Non. Mech. 35 (2000) 1059–1065.

[30] X.J. Zheng, X.Z. Wang, Analysis of magnetoelastic interaction of rectangular ferromagnetic plates with nonlinear magnetization, In. Solids Struc. 38 (2001) 8641–8652.

[31] G.Y. Baghdasaryan, Z.N. Danoyan, M.A. Mikilyan , Issues of dynamics of conductive plate in a longitudinal magnetic field, In. J. Solids Struc. 50 (2013) 3339–3345.

[32] X. Wang, J.S. Lee, X. Zheng, Magneto-thermo-elastic instability of ferromagnetic plates in thermal and magnetic fields, In. J. Solids Struc. 40 (2003) 6125–6142

[33] G. Y. Wu, Transient vibration analysis of a pinned beam with transverse magnetic fields and thermal loads, J. Vib. Acoust. 127 (2005) 247–253.

[34] G. Y Wu, The analysis of dynamic instability on the large amplitude vibrations of a beam with transverse magnetic fields and thermal load, J. Sound Vib. 302 (2007) 167–177.

[35] G.Y. Wu, 2013, Non-linear vibration of bimaterial magneto-elastic cantilever beam with thermal loading, In. J. Non. Mech. 55 (2013) 10–18.

[36] Z. Jia, W. Liu, Y. Zhang, F. Wang, D. Guo, A nonlinear magnetomechanical coupling model of giant magnetostrictive thin films at low magnetic fields, Sen. Act. A 128 (2006) 158–164.

[37] Anandkumar R. Annigeri, N. Ganesan, S. Swarnamani, Free vibration behaviour of multiphase and layered magneto-electro-elastic beam, J. Sound Vib. 299 (2007) 44–63.

[38] G.Y. Zhou, Q.Wang, Magnetorheological elastomer-based smart sandwich beam with nonconductive skins, Smart Mat. Struc. 14 (2005) 1001–1009.

[39] G.Y. Zhou, Q. Wang, Use of magnetorheological elastomer in an adaptive sandwich beam with conductive skins. Part I: magnetoelasticloadsin conductive skins, In. J. Solids Struc. 43(2006) 5386–5402.

[40] G.Y. Zhou, Q. Wang, Use of magnetorheological elastomer in an adaptive sandwich beam with conductive skins. PartII: dynamic properties, In. J. Solids Struc. 43(2006) 5403–5420.

[41] B. Nayak, S.K. Dwivedy, K.S.R.K. Murthy, Dynamic analysis of magnetorheological elastomer-based sandwich beam with conductive skins under various boundary conditions, J. Sound Vib. 330 (2011) 1837–1859.

[42] B. Nayak, S.K. Dwivedy , K.S.R.K. Murthy, Dynamic stability of a rotating sandwich beam with magnetorheological elastomer core, Euro. J. Mech. A/Solids 47 (2014) 143–155

[43] B. Nayak, S.K. Dwivedy, K.S.R.K. Murthy, Multi-frequency excitation of magnetorheological elastomer-based sandwich beam with conductive skins, In. J. Non. Mech. 47 (2012) 448–460.

[44] T. Kong, D.X. Li, X. Wang, Thermo-magneto-dynamic stresses and perturbation of magnetic field vector in a non-homogeneous hollow cylinder , Appl. Math. Mod. 33 (2009) 2939–2950.

[45] L. Wang, W.B. Liu, H.-L. Dai, Dynamics and instability of current-carrying microbeams in a longitudinal magnetic field, Phy. E 66 (2015) 87–92.

[46] K. Kiani, Surface effect on free transverse vibrations and dynamic instability of current-carrying nanowires in the presence of alongitudinal magnetic field, Phy. Let. A 378 (2014) 1834–1840.

[47] K. Kiani, Stability and vibrations of doubly parallel current-carrying nanowires immersed in a longitudinal magnetic field, Phy. Let. A 379 (2015) 348–360.